**Question 1.** Determine whether the lines given by the equations below are parallel, perpendicular, or neither. Also, find a rigorous algebraic solution for each problem.

**a.     b.       c.**

**Solution:**

**a.**

**y = m(x) + b** (Slope Intercept form)

**First part:**3y + 4x = 12

y = (12 – 4x)/3

y = -4x/3+4

**Now second part:**

-6y = 8x + 1

y = (8x + 1)/-6

y = -4x/3-1/6

Both part have same slope **(−4/ 3**) mean line are **parallel**

**b.**

**First part:**  
3y + x = 12

y = (12 – x)/3

y = -x/3+4

**Now second part:**

**−** y = 8x + 1

y = -8x – 1

slopes are different **(−1/3​ and −8**), so lines are **neither parallel nor perpendicular.**

**c.**

**First part:**  
4x – 7y = 10

y = (10 –4x)/-7

y = 4x/7 – 10/7

**Now second part:**

7x + 4y = 1

y = (1 - 7x) / 4

y = -7x/4 + 1/4

slopes are different (**4/7 and -7/4**) and **opposite to each other**, so lines are **perpendicular** to each other.

**Question 2.** A ball is thrown in the air from the top of a building. Its height, in meters above ground, as a function of time, in seconds, is given by [ h(t)=-4.9t^2+24t+8 ](https://my.uopeople.edu/filter/tex/displaytex.php?texexp=%20h(t)%3D-4.9t%5e2%2B24t%2B8%20). What is the height of the building? What is the maximum height reached by the ball? How long does it take to reach maximum height? Also, find a rigorous algebraic solution for the problem.

**Solution:**

**h(t) = -4.9t2 + 24t + 8 --------------------(1)**

height of the building represent by the constant term in the function  
so, height of the building = **8 meters**

The formula for finding the x-coordinate of the vertex (*t*) in a quadratic function **ax2 + bx + c** is given by **t = -b/2a**

h(t) = -4.9t2 + 24t + 8

here a = -4.9 and b = 24

putting these values, we get

t = -24/ (2(-4.9)) => -24/-9.8

**t = 2.45 sec** (this is the time where the ball will reach to its maximum height)

putting value of t in equation 1, we get

h(2.45) = -2.9(2.45)2 + 24(2.45) + 8  
h(2.45) = -2.9(6.0025) + 58.8 + 8

**h(2.45) = 37.388 meters**

**Question 3.** In a market, there are 100 shops, and a shop’s average per day income is $200. If one more shop is added to the market, the average income per shop reduces by $5 due to price competition. Conversely, the decrease of one shop can increase the average income per shop by $5 due to cartelization among the shopkeepers. Please suggest the optimal number of shops to the city’s revenue department from a total income (revenue) point of view of the market?

**Solution:**

Per day average income of shop = **200$** when there are **100** shops

So total income for the revenue department for **100** shops will be given by:

**R100 = 100 x 200** Where R100 represents Rupees for **100** shops

**R100 = 20,000$**

If **1** shop added, average income decrease by **5$,**  
So by calculating total income for the revenue department for **101** shops becomes:

**R101 = 101 x 195** (because price decrease by **5$)**

**R101 = 19,695$**  
  
And If **1** shop removed, average income will decrease by **5$,** then  
**R99 = 99 x 205** (because price increase by **5$)**

**R99 = 20,295$**  
  
Now comparing these calculation, **99** **shops** should be in the **market**.

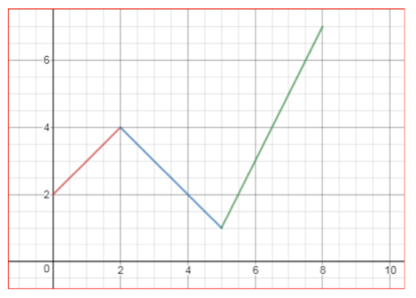
**Questions 4**. Lines can be used to approximate a wide variety of functions; often a function can be described using many lines. If a stock price goes from $10 to $12 from January 1st to January 31; from $12 to $9 from February 1st to February 28th; and from $9 to $15 from March 1st to March 31th. Is the price change of the stock $10 to $15 from January 1st to March 31st a straight line? It is clear that in each of the three time intervals mentioned there was a complex daily variation of prices as in an electrocardiogram. But what would be a simplified solution for a first naive view of the situation? Would a simple function hold up? What is the simplest function to represent this situation? Does your naïve initial and simplified model allow you to predict the behavior of the stock in the next month?

How can I use three “pieces” of lines to describe the price movements from the beginning of January to the end of March? Show the graph for the price movement.

y = x + 2    {0 < x < 2}

y = –x + 6  {2 < x < 5}

y = 2x – 9  {5 < x < 8}



**Solution:**

**From January 1to January 31,**

**p** varies from10 to 12.

Determining the equation of line describing the variation of prices

The line passes through **(1,10), (31,12)**

=

= =>

15p – 150 = d – 1

**15p – d = 149**

**From February 1 to February 28,**

**p** varies from 12 to 9.

The line passes through **(1,12), (28,9)**

=

= =>

9p – 108 = 1 – d

**9p + d = 109**

**From March 1 to March 31,**

p varies from 9 to 15.   
The line passes through **(1,9), (31,15)**

=

= =>

5p – 45 = d – 1

**5p – d = 44**

**When p = 10**

5(10) – d = 44

50 – d = 44

**d = 6**

**When p = 15**

5(15) – d = 44

75 – d = 44

**d = 31**

The price changes from 10 to 15, **is not a straight line**

what would be a simplified solution for a first naive view of the situation?

**15p – d = 149 (Linear equation)**

Would a simple function hold up?

**Yes**

What is the simplest function to represent this situation?

**15p – d = 149**

Does your naïve initial and simplified model allow you to predict the behavior of the stock in the next month?

**Yes**